$$2(x-r)+2(mx-r)+2r=x+mx+x\sqrt{(1+m^2)}.$$

$\therefore\ r=\frac{1}{2}x[1+m-\sqrt{(1+m^2)}]$.

$GH=\frac{1}{3}BC=\frac{1}{3}mx$, $KH=\frac{1}{2}AB-HB=\frac{1}{2}x-r$.

$\therefore\ KH=\frac{1}{2}x[\sqrt{(1+m^2)}-m)]$, $GK=\frac{1}{3}CK=\frac{1}{3}x\sqrt{(\frac{1}{4}+m^2)}$.

$\therefore\ KG^2=GH^2+KH^2$, or $\frac{1}{9}x^2(\frac{1}{4}+m^2)=\frac{1}{9}m^2x^2+\frac{1}{4}x^2[\sqrt{(1+m^2)}-m]^2$.

$\therefore\ \frac{1}{4}[\sqrt{(1+m^2)}-m]^2=\frac{1}{36}$, or $\sqrt{(1+m^2)}-m=\frac{1}{3}$. $\therefore\ m=\frac{4}{3}$.

$\therefore\ AB : BC=3 : 4$, and $HB : AB : BC : AC=1 : 3 : 4 : 5$.

Also solved by *J. C. NAGLE*.

DIOPHANTINE ANALYSIS.

60. Proposed by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

It is required to find six positive numbers, such that if each be diminished by five-half times the fifth power of their sum the six remainders will be rational fifth powers.

Solution by the PROPOSER.

Let u, v, w, x, y, z, be the six numbers required, and let $u+v+w+x+y+z=s$.

Then $u-\frac{5}{2}s^5=h^5s^5/q^5$, $v-\frac{5}{2}s^5=k^5s^5/q^5$, $w-\frac{5}{2}s^5=l^5s^5/q^5$,

$x-\frac{5}{2}s^5=m^5s^5/q^5$, $y-\frac{5}{2}s^5=n^5s^5/q^5$, $z-\frac{5}{2}s^5=p^5s^5/q^5$,

Adding these six equations we get

$$s-15s^5=(s^5/q^5)(h^5+k^5+l^5+m^5+n^5+p^5).$$

Let $h^5+k^5+l^5+m^5+n^5+p^5=q^5$. $\therefore\ s=\frac{1}{2}$.

$\therefore\ u=\frac{1}{32}[\frac{5}{2}+(h^5/q^5)]$, $v=\frac{1}{32}[\frac{5}{2}+(k^5/q^5)]$, $w=\frac{1}{32}[\frac{5}{2}+(l^5/q^5)]$,

$x=\frac{1}{32}[\frac{5}{2}+(m^5/q^5)]$, $y=\frac{1}{32}[\frac{5}{2}+(n^5/q^5)]$, $z=\frac{1}{32}[\frac{5}{2}+(p^5/q^5)]$.

Let $h=4$, $k=5$, $l=6$, $m=7$, $n=9$, $p=11$, $q=12$.

$u=\frac{121717}{155552}$, $v=\frac{625520524}{796262624}$, $w=\frac{81}{1024}$, $x=\frac{638887}{796262627}$, $y=\frac{9803}{32768}$, $z=\frac{7831131}{796262624}$.

Let $h=5$, $k=10$, $l=11$, $m=16$, $n=19$, $p=29$, $q=30$.

$u=\frac{1944417}{2488832}$, $v=\frac{121717}{155552}$, $w=\frac{609111051}{777600000}$, $x=\frac{3862411}{4860000}$, $y=\frac{63222609}{77760000}$, $z=\frac{81261149}{77760000}$.

AVERAGE AND PROBABILITY.

58. Proposed by HENRY HEATON, M. Sc., Atlantic, Iowa.

From a point on the surface of a circle two lines are drawn to the circumference. Required the average area that may be cut from the circle in this way if the lines are supposed to be drawn at equal angular intervals.

Query I. How does this differ from problem 32?

Query II. Is *sector* the proper word to use for the surface thus cut off?

Query III. Is it absolutely correct to use the word *random* in average problems?